EFFINET

Efficient Integrated Real-time Monitoring and Control of Drinking Water Networks

Economic and Technical Evaluation of Operational Management Algorithms

Deliverable 2.3

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Economic and technical evaluation of operational management algorithms

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Abstract:

This deliverable report elaborates on the technical feasibility of the proposed optimal operational control scheme as introduced in deliverable D2.1 and the proposed control and optimisation algorithms presented in D2.2, while it also studies the economic benefits that the proposed control methodology is expected to bring about.

We also address the management of drinking water networks (DWNs) regarding a multi-objective cost function by means of economically-oriented model predictive control (EMPC) strategies. Specifically, assuming the water demand and the energy price as periodically time-varying signals, this report shows that the EMPC framework is flexible to enhance the control of DWNs without relying on hierarchical control schemes that require the use of real-time optimisers (RTO) or steady-state target optimisers (SSTO) in an upper layer. Four different MPC strategies are discussed: a hierarchical two layer approach, a standard EMPC where the multi-objective cost function is optimized directly, and two different modifications of the latter, which are meant to overcome possible feasibility losses in the presence of changing operating patterns. The discussed schemes are tested and compared by means of a case study taken from a part of the Barcelona DWN which is the main case study of the EFFINET project.

The results presented in this report have also been disseminated in scientific conferences such as [16, 10] and [15].

Contents

Chapter 1

Technical and Economic Assessment

1.1 Introduction

The management and control of water systems is a challenging problem due to different sources of uncertainty. The availability of historical data allows to accurately predict the behaviour of the system disturbances over large horizons but still a meaningful degree of uncertainty is present. Besides uncertainty, water systems often exhibit other complicating features such as multi-variable interactions, transport delays, hard constraints on the system variables, or multiple conflicting control goals. Due to all these problems, model predictive control (MPC) has been proposed as a suitable technique to control water systems, see a further discussion in [24]. By using MPC, it is possible to explicitly incorporate the aforementioned features into an optimisation problem in a systematic manner. To this end, a mathematical model of the system is used to predict its future behaviour, which is optimised in a receding horizon fashion according to a given performance index over a certain prediction horizon [22].

This deliverable report focuses on the way that uncertainty can be faced from an MPC framework in the control of drinking water networks (DWN). The simplest way to do this is by ignoring the explicit influence of disturbances or using their expected value. Unfortunately, this approach may lead to poor control performance or frequent constraints violation. As discussed in [6], alternative approaches of MPC for stochastic systems are based on *min-max* MPC, *tube-based* MPC, and *stochastic* MPC. The first two consider disturbances to be unmeasured but bounded in a predefined set, which is more conservative and reduces the control performance due to the worst-case nature of the schemes. On the other hand, the stochastic MPC considers a more realistic description of uncertainty, which leads to less conservative control approaches at the expense of a more complex modelling of the disturbances. The stochastic approach has a mature theory in the field of optimisation [5], but a renewed attention has been given to the stochastic programming techniques as powerful tools for control design.

From the wide range of stochastic MPC methods, this report focuses on *tree-based MPC* (TB-MPC) and *chance-constrained MPC* (CC-MPC). Regarding TB-MPC, see, e.g., [28] and [21], uncertainty is addressed by considering simultaneously a set of possible disturbance scenarios modelled as a rooted tree, which branches along the prediction horizon. On the other hand, CC-MPC citeSchwarm1999 is a stochastic control strategy that describes robustness in terms of probabilistic (chance) constraints, which require that the probability of violation of any operational requirement or physical constraint is below a prescribed value, representing the notion of reliability or risk of the system. By setting this value properly, the operator/user can trade conservatism against performance. Relevant works that address the CC-MPC approach in water systems can be found in [14, 25] and references therein.

The main contribution of this report consists in the design and assessment of TB-MPC and CC-MPC controllers applied for the operational management of a drinking water network (DWN), discussing their advantages and weakness in the sense of applicability and performance. The particular case study is related to the DWN of Barcelona (Spain).

The remainder of the chapter is organised as follows: Section 1.2 states the control problem, describing its formulation within both TB-MPC and CC-MPC frameworks addressed here. Section 1.3 describes the case study consisting in a small DWN, where the proposed approaches are evaluated. Section 1.4 highlights the concluding remarks and some future research directions.

1.2 Stochastic MPC Formulations

1.2.1 Problem Statement

MPC uses a mathematical model to calculate the optimal control actions according to a given cost function [22]. It is assumed that the system behaviour can be described at each time instant $k \in \mathbb{Z}$ by the following discrete-time difference equation:

$$x_{k+1} = Ax_k + Bu_k + Ew_k, \tag{1.1}$$

where $x_k \in \mathbb{R}^{n_x}$ is the state of the system, $u \in \mathbb{R}^{n_u}$ is the vector of manipulated variables, and $w \in \mathbb{R}^{n_w}$ is a vector of measurable disturbances. Moreover, $A, B,$ and E are time-invariant matrices of proper dimensions. It is also considered that the system is subject to hard state and input constraints, which can be posed as

$$x \in \mathcal{X} \triangleq \{x_k \in \mathbb{R}^{n_x} | \; Gx_k \leq g, \; \forall k\}, \tag{1.2a}$$

$$u \in \mathcal{U} \triangleq \{u_k \in \mathbb{R}^{n_u} | \; Fu_k \leq f, \; \forall k\}, \tag{1.2b}$$

where $\mathcal{X}$ and $\mathcal{U}$ are closed polyhedra defined by a system of linear inequalities. In this sense, $G \in \mathbb{R}^{c_x \times n_x}$, $g \in \mathbb{R}^{c_x}$, $F \in \mathbb{R}^{c_u \times n_u}$, $f \in \mathbb{R}^{c_u}$, being c_x and c_u the number of state and input constraints, respectively.

The control goal is to minimise a convex (possible multi-objective) cost function $\ell(x, u) : \mathcal{X} \times \mathcal{U} \to \mathbb{R}$, which might bear any functional relationship to the operating cost of the system. From the model in (1.1), let $w_{[k:k+N-1]} \triangleq (w_k, w_{k+1|k}, \dots, w_{k+N-1|k})$ be the sequence of disturbances over a fixed time prediction horizon $N \in \mathbb{Z}_+$. The first element of the sequence is measured, while the rest of the elements, i.e., $w_{k+i|k}$, denote estimates of future disturbances computed by an exogenous system and available at each time instant k. Hence, the MPC controller design is based on the solution of the following finite horizon optimisation problem (FHOP):

$$\min_{\bar{u}_{[0:N-1]}} \sum_{i=0}^{N-1} \ell(\bar{x}_i, \bar{u}_i) \tag{1.3a}$$

subject to:

$$\bar{x}_{i+1} = A\bar{x}_i + B\bar{u}_i + E\bar{w}_i, \tag{1.3b}$$
$$\bar{x}_{i+1} \in \mathcal{X}, \tag{1.3c}$$
$$\bar{u}_i \in \mathcal{U}, \tag{1.3d}$$
$$\bar{x}_0 = x_k, \quad \bar{w}_0 = w_k, \tag{1.3e}$$
$$\bar{w}_i = w_{k+i|k} \quad \forall i \in \mathbb{Z}_1^{N-1}, \tag{1.3f}$$

where vectors $\bar{x}$,$\bar{u}$, and $\bar{w}$ denote the predicted value of the states, inputs and measured disturbances at the prediction step i. Notation $\mathbb{Z}_a^b$ expresses the set of integer numbers from a to b, both limits included, i.e., $\{a, a+1, \dots, b\}$.

Assuming that (1.3) is feasible, i.e., there exists a non-empty solution given by the optimal sequence of control inputs $\bar{u}^*_{[0:N-1]} \triangleq (\bar{u}^*_0, \bar{u}^*_1, \dots, \bar{u}^*_{N-1})$, then the receding horizon philosophy commands to apply the control action

$$u_k = \kappa_{\mathrm{mpc}}(x_k, w_{[k:k+N-1]}) = \bar{u}^*_0, \tag{1.4}$$

and disregards the rest of the sequence of the predicted manipulated variables. At the next time instant k, the FHOP (1.3) is solved again using the current measurements of states and disturbances and the most recent forecast of these latter over the next future horizon.

Given the stochastic nature of future disturbances, the prediction model (1.3b) involves exogenous additive uncertainty, hence, the compliance of constraints for a given control input cannot be ensured. This means that, even if the predictive controller finds a feasible solution to achieve the operational goals, there is a certain probability that real outputs may violate constraints. Therefore, the use of SP-MPC strategies may allow to establish a trade-off between robustness and performance.

1.2.2 Chance-Constrained Model Predictive Control

As pointed out before, the optimal solution to (1.3) does not always imply feasibility of the real system due to the stochastic nature of disturbances. Therefore, it is appropriate to relax the original constraints in (1.2a) with probabilistic statements in the form of *chance constraints*. In this way, state constraints are required to be satisfied with a predefined probability to manage the reliability of the system as follows:

$$\mathbb{P}\left[x_k \in \mathcal{X}\right] \geq 1 - \delta_x, \quad \forall k \in \mathbb{Z}_+, \tag{1.5}$$

where $\mathbb{P}$ denotes the probability operator and $\delta_x \in (0,1)$ is the *risk acceptability level* of constraint violation for the states. Let (1.5) be rewritten as

$$\mathbb{P}\left[G_{(j)}x_k \leq g_{(j)},\ \forall j \in \mathbb{Z}_1^{c_x}\right] \geq 1 - \delta_x, \quad \forall k \in \mathbb{Z}_+, \tag{1.6}$$

where $G_{(j)}$ and $g_{(j)}$ denote the j^{th} row of G and g, respectively. Hence, this probabilistic constraint is in the form of the so called *joint chance constraint*, which requires that the c_x element-wise inequalities have to be jointly fulfilled with the given probability at each time instant i.

In general, joint chance constraints lack from analytic expressions due to the involved multivariate probability distribution. Nevertheless, sampling-based methods, numeric integration, and convex analytic approximations exists, see e.g., [5] and references therein. Here, (1.6) is approximated following the results in [27] and [23] by upper bounding the joint constraint and assuming a uniform distribution of the joint risk among a set of *individual chance constraints* that are later transformed into equivalent deterministic constraints under the following consideration:

Assumption 1. *Each disturbance in w follows a log-concave univariate distribution, which stochastic description is known.*

Given the dynamic model in (1.1), the stochastic nature of the disturbances w makes the states x to be also stochastic variables. Then, it follows for all time instants that

$$F_{Gx}(g) = \mathbb{P}\left[\left\{G_{(1)}x \leq g_{(1)}, \ldots, G_{(c_x)}x \leq g_{(c_x)}\right\}\right]. \tag{1.7}$$

Defining the events $C_j \triangleq \left\{G_{(j)}x \leq g_{(j)},\right\}$, $\forall j \in \mathbb{Z}_1^{c_x}$, and their complements $C_j^c \triangleq \left\{G_{(j)}x > g_{(j)},\right\}$, then it follows that

$$F_{Gx}(g) = 1 - \mathbb{P}\left[(C_1^c \cup \ldots \cup C_{c_x}^c)\right] \geq 1 - \delta_x. \tag{1.8}$$

Taking advantage of the *union bound*, the Boole's inequality allows to bound the probability of the second term in the left-hand side of (1.8), stating that the probability that at least one event happens is no longer than the sum of the individual probabilities [27]. Hence, it follows that

$$\sum_{j=1}^{c_x} \mathbb{P}\left[C_j^c\right] \leq \delta_x \Leftrightarrow \sum_{j=1}^{c_x} \left(1 - \mathbb{P}\left[C_j\right]\right) \leq \delta_x. \tag{1.9}$$

At this point, a set of constraints arise from previous result as sufficient conditions to enforce the joint chance constraint (1.6), by allocating the joint risk δ_x in c_x separate risks $\delta_{x,j}$, $j \in \mathbb{Z}_1^{c_x}$. These constraints are:

$$\mathbb{P}\left[C_j\right] \geq 1 - \delta_{x,j}, \quad \forall j \in \mathbb{Z}_1^{c_x}, \tag{1.10}$$

$$\sum_{j=1}^{c_x} \delta_{x,j} \leq \delta_x, \tag{1.11}$$

$$0 \leq \delta_{x,j} \leq 1, \tag{1.12}$$

where (1.10) forms the set of c_x resultant individual chance constraints, which bounds the probability that each inequality of the receding horizon problem may fail; and (1.11) and (1.12) are conditions imposed to bound the new single risks in such a way that the joint risk bound is not violated. Any solution that satisfies the above constraints, is guaranteed to satisfy (1.6). Assigning, e.g., a fixed and equal value of risk to each individual constraint, i.e., $\delta_{x,j} = \delta_x / c_x$, satisfies (1.11) and (1.12) [23].

After decomposing the joint constraints into a set of individual constraints, the *deterministic equivalent* of each separate constraint may be used given that the probabilistic statements are not suitable for algebraic solution. Such deterministic equivalents might be obtained following [9]. Assuming a known (or approximated) quasi-concave probabilistic distribution $F_{Ew}(b) = \mathbb{P}[Ew \leq b]$ for the effect of the stochastic disturbance in the dynamic model (1.1), then it follows that

$$\begin{aligned} &\mathbb{P}\left[G_{(j)} x_{k+1} \leq g_{(j)}\right] \geq 1 - \delta_{x,j} \\ &\Leftrightarrow G_{(j)}\left(A x_k + B u_k\right) \leq g_{(j)} - z_{\delta_{x,j}}, \end{aligned} \tag{1.13}$$

where $z_{\delta_{x,j}} \triangleq F^{-1}_{G_{(j)} E w_k}(1 - \delta_x / c_x)$ is the quantile function of $G_{(j)} E w_k$. Given that the state x is also a stochastic variable, the above deterministic equivalent should be considered in terms of the expectation of the variables. In this way, the original constrained set $\mathcal{X}$ is contracted with the effect of the c_x deterministic equivalents and replaced in (1.3c) with the stochastic feasibility set $\mathcal{X}_s$ given by

$$\begin{aligned} \mathcal{X}_s \triangleq \; &\{\bar{x} \in \mathbb{R}^{n_x} \mid \exists u \in \mathcal{U}, \\ &G_{(j)}\left(A \bar{x}_k + B \bar{u}_k\right) \leq g_{(j)} - z_{\delta_{x,j}}\}, \; \forall j \in \mathbb{Z}_1^{c_x}. \end{aligned} \tag{1.14}$$

The reformulated predictive controller solves the following deterministic equivalent FHOP for the expectation $\mathbb{E}[\cdot]$ of the cost function in (1.3a):

$$\min_{\bar{u}(0:N-1)} \sum_{i=0}^{N-1} \mathbb{E}\left[\ell(\bar{x}_i, \bar{u}_i)\right] \tag{1.15a}$$

subject to: (1.3b), (1.3d), (1.3e), (1.3f),

$$\bar{x}_{i+1} \in \mathcal{X}_s. \tag{1.15b}$$

1.2.3 Tree-based Model Predictive Control

The deterministic equivalent CC-MPC proposed before might be still conservative if the probabilistic distributions of the stochastic variables are not well characterized or do not lie in a log-concave form. Therefore, this section presents a TB-MPC strategy that relies in scenario-trees to approximate the original problem, dropping Assumption 1. The approach followed by TB-MPC is based on modelling the possible scenarios of the disturbances as a rooted tree. This means that all the scenarios start from the same expected disturbance value. From that point, the scenarios must remain equal until the point in which they diverge from each other, which is called a bifurcation point. For instance, consider two scenarios a and b defined by the sequences $w^a_{[0:N-1]}$ and $w^b_{[0:N-1]}$ and let k_{bp} be the moment in which they diverge, then $w^a_k = w^b_k$ for $k \leq k_{\mathrm{bp}}$ and $w^a_k \neq w^b_k$ for $k > k_{\mathrm{bp}}$.

Notice that before a bifurcation point, the evolution followed by the disturbance cannot be anticipated because different evolutions are possible. For this reason, the controller has to calculate control actions that are valid for all the scenarios in the branch. Once the bifurcation point has been reached, the uncertainty is solved and the controller can calculate specific control actions for the scenarios in each of the new branches. Hence, the outcome of TB-MPC is not a single sequence of control actions, but a tree with the same structure of that of the disturbances. As in standard MPC, only the first element of this tree is applied (the root) and the problem is repeated in a receding horizon fashion.

The easiest way to understand the optimisation problem that has to be solved in TB-MPC is to solve as many instances of Problem (1.3) as the number N_s of scenarios considered. Due to the increasing uncertainty, it is necessary to include non-anticipativity constraints [30] in the MPC formulation so that the calculated input sequence is always ready to face any possible future bifurcation in the tree. More specifically, if $u^a_{[0:N-1]}$ and $u^b_{[0:N-1]}$ are the input sequences that correspond respectively to the scenarios a and b calculated according to Problem (1.3), then the non-anticipativity constraint $u^a_k = u^b_k$ has to be satisfied whenever $w^a_k = w^b_k$ in order to guarantee that the set of inputs $u^j_{[0:N-1]}$ with $j \in \mathbb{Z}_1^{N_s}$ forms a tree with the same structure of that of the disturbances.

In this way, the TB-MPC controller has to solve the following FHOP, accounting for the $N_s \in \mathbb{Z}_+$ scenarios, each with probability $p_j \in [0,1]$ and $\sum_{j=1}^{N_s} p_j = 1$:

$$\min_{\bar{u}^j_{[0:N-1]}} \sum_{j=1}^{N_s} \left(p_j \sum_{i=0}^{N-1} \ell(\bar{x}^j_i, \bar{u}^j_i) \right) \tag{1.16a}$$

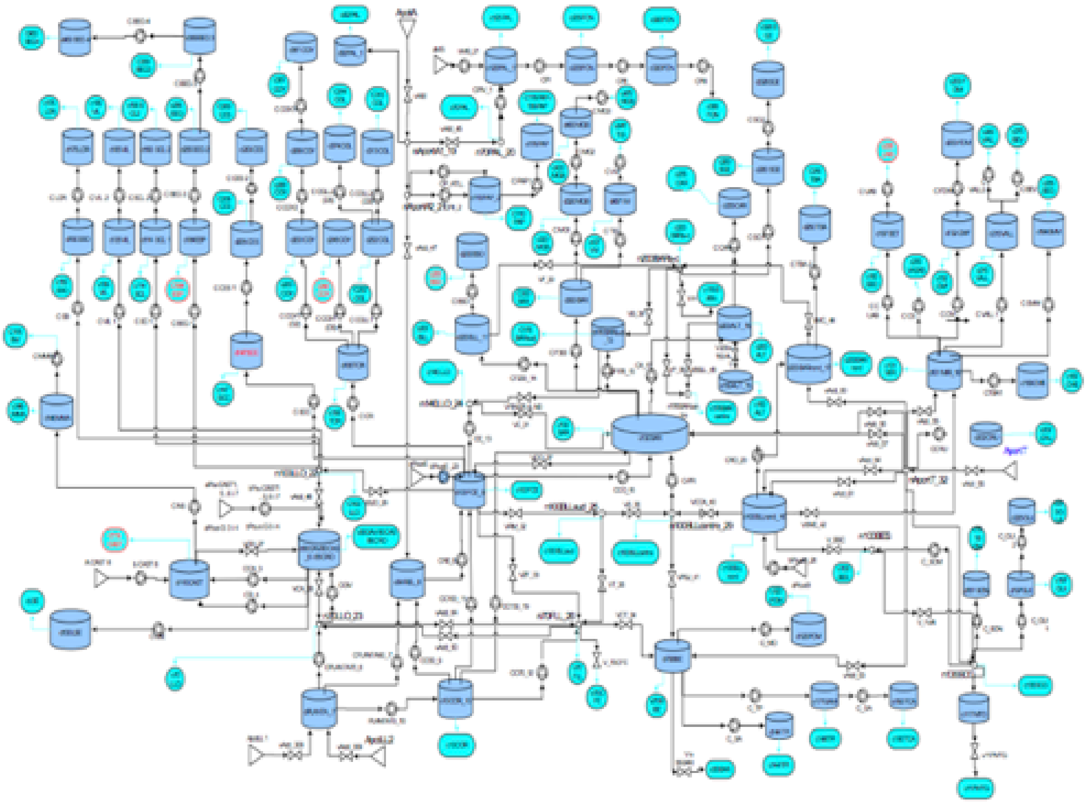

Figure 1.1: Topology of the large-scale DWN of Barcelona.

subject to:

$$\bar{x}^j_{i+1} = A\bar{x}^j_i + B\bar{u}^j_i + E\bar{w}^j_i, \tag{1.16b}$$

$$\bar{x}^j_{i+1} \in \mathcal{X}, \tag{1.16c}$$

$$\bar{u}^j_i \in \mathcal{U}, \tag{1.16d}$$

$$\bar{x}^j_0 = x_k, \quad \bar{w}^j_0 = w_k, \tag{1.16e}$$

$$\bar{w}^j_i = w^j_{k+i|k}, \quad \forall i \in \mathbb{Z}^{N-1}_1, \tag{1.16f}$$

$$\bar{u}^a_i = \bar{u}^b_i \quad \text{if} \quad \bar{w}^a_i = \bar{w}^b_i, \quad \forall a, b \in \mathbb{Z}^{N_s}_1. \tag{1.16g}$$

Remark 1. *The number of scenarios used to build the rooted tree should be determined regarding the computational capacity and the probability of risk that the manager is willing to accept.* ◊

1.3 A drinking water network case study

1.3.1 Case Study Description

This section briefly describes a motivational example useful to assess SP-MPC approaches to solve multi-objective control problems attained to DWNs subject to stochastic disturbances and constraints. The case study consists of a small but representative example of a DWN that has to supply certain water demands by making optimal use of water sources and network components in order to minimise economic costs and guarantee service reliability. In general, the DWN operation is driven by the electricity prices and the exogenous and endogenous demands. The system under study is a portion extracted from the Barcelona DWN reported in [24].

1.3.2 Control-oriented Model

Consider a directed graph abstracted from Fig. 1.1. Setting the volume in storage tanks as the state variables $x \in \mathbb{R}^{n_x}$, the flow through the actuators as the manipulated inputs $u \in \mathbb{R}^{n_u}$, and the demanded flows as *additive* measured disturbances $d \in \mathbb{R}^{n_d}$, then the control-oriented model of the network may be abstracted from the connectivity analysis and described by the following set of discrete difference-algebraic equations for all time instant $k \in \mathbb{Z}_+$:

$$x_{k+1} = Ax_k + Bu_k + B_p d_k, \quad (1.17a)$$
$$0 = E_u u_k + E_d d_k, \quad (1.17b)$$

where (2.1a) and (2.1b) describe the mass balance equations for storage tanks and intersection nodes, respectively. Moreover, A, B, B_p, E_u and E_d, are time-invariant matrices of dimensions dictated by the network topology.

Assumption 2. *The states in x and the demands in d are observable at time k, and the pair (A, B) is controllable.*

Assumption 3. *The realisation of disturbances at the current time instant k may be decomposed as*

$$d_k = \bar{d}_k + e_k, \quad (1.18)$$

where $\bar{d}_k \in \mathbb{R}^{n_d}$ is the vector of expected disturbances, and $e_k \in \mathbb{R}^{n_d}$ is the vector of forecasting errors with non-stationary uncertainty and a known (or approximated) quasi-concave probability distribution $\mathcal{D}$. Therefore, the stochastic nature of each j^{th} row of d_k is described by $d_{(j),k} \sim \mathcal{D}_i(\bar{d}_{(j),k}, \Sigma(e_{(j),k}))$, where $\bar{d}_{(j),k}$ denotes its mean, and $\Sigma(e_{(j),k})$ its variance.

The system is subject to hard state and input constraints given by the following convex and closed polytopic sets:

$$\mathcal{X} \triangleq \{x_k \in \mathbb{R}^{n_x} | x_{\min} \leq x_k \leq x_{\max}\}, \quad \forall k, \quad (1.19a)$$
$$\mathcal{U} \triangleq \{u_k \in \mathbb{R}^{n_u} | u_{\min} \leq u_k \leq u_{\max}\}, \quad \forall k, \quad (1.19b)$$

where $x_{\min} \in \mathbb{R}^{n_x}$ and $x_{\max} \in \mathbb{R}^{n_x}$ denote the vectors of minimum and maximum volume capacities in tanks, respectively, given in m^3; while $u_{\min} \in \mathbb{R}^{n_u}$ and $u_{\max} \in \mathbb{R}^{n_u}$ denote the vectors of minimum and maximum flow capacities through the system actuators, respectively, given in m^3/s. Moreover, for safety and supply service reliability, the states are subject to the following management soft constraint:

$$x_k \geq s_k - \xi_k \geq 0, \quad \forall k, \tag{1.20}$$

where $s_k \in \mathbb{R}_+^{n_x}$ is a positive vector of base stocks (minimal volume in each tank to avoid stock-outs) and $\xi_k \in \mathbb{R}_+^{n_x}$ is a vector of positive slack variables to be minimised, which represent the amount of water volume in tanks that is allowed to go below the desired base stocks.

1.3.3 Demand Modelling and Scenario Generation

In DWNs, the uncertainty is generally introduced by the unpredictable behaviour of water consumers. Therefore, a proper demand modelling is required to achieve an acceptable water supply service level. For our case study, time series forecasting based on auto-regressive integrated moving average (ARIMA) models is used due to its ability to capture complex linear dynamics from historical data [3]. Once these models are calibrated, they are used here to generate a large number of possible demand scenarios by Monte Carlo sampling for a given prediction horizon $N \in \mathbb{Z}_+$. For the CC-MPC approach the mean demand path is used, while for the TB-MPC approach a set of scenarios is selected. A large number of scenarios might improve the robustness of the TB-MPC approach but at the cost of additional computational burden and economic performance losses. Hence, a trade-off must be achieved between performance and computational burden. To this end, a representative subset of scenarios may be chosen using scenario reduction algorithms. In report D2.3, the backward reduction algorithm in [17] is used to reduce a specified initial fan of N_s equally probable scenarios into a rooted tree of N_r scenarios.

1.3.4 Control Problem Formulation

The formulation of the SP-MPC problems for the above case study should address the design of a control law $u_k = \kappa\left(x_k, d_{[k:k+N-1]}\right)$ that minimises, in a receding horizon fashion, the following cost function:

$$\ell_k \triangleq \lambda_1 \ell^e(u_k, \alpha_k) + \lambda_2 \ell^s(\xi_k) + \lambda_3 \ell^\Delta(\Delta u_k), \tag{1.21}$$

where $\ell^e(u_k, \alpha_k) \triangleq \alpha_k^\top W_e u_k \in \mathbb{R}_{\geq 0}$ represents the economic cost of network operation that depends on a time-of-use pricing scheme given by $\alpha_k \triangleq \left(\alpha_1 + \alpha_{2,k}\right) \in \mathbb{R}^{n_u}$, which takes into account a fixed water production cost $\alpha_1 \in \mathbb{R}^{n_u}$ and a time-varying water pumping cost $\alpha_{2,k} \in \mathbb{R}^{n_u}$ that changes according to the electric tariff; $\ell^s(\xi_k) \triangleq \xi_k^\top W_s \xi_k \in \mathbb{R}_{\geq 0}$ is a performance index that penalises the amount of volume ξ going below from the

Table 1.1: Assessment of the CC-MPC and TB-MPC applied to the DWN case study.

CC-MPC					TB-MPC					
δ	Φ_1	Φ_2	Φ_3	Φ_4	Φ_1	Φ_2	Φ_3	Φ_4	N_r	N_s
					58397.14	0	0	0.940	5	
0.3	58535.80	0	0	1.246	58280.69	1	0.510	1.607	10	19
					58279.95	1	4.155	2.373	14	
					58482.14	3	0.183	1.178	7	
0.2	58541.19	0	0	1.208	58903.63	0	0	2.326	14	29
					58452.41	0	0	4.048	21	
					58610.32	0	0	2.570	14	
0.1	58558.29	0	0	1.252	58630.20	0	0	6.655	29	59
					58656.56	1	0.178	13.466	44	
					-	-	-	-	149	
0.01	58612.28	0	0	1.253	-	-	-	-	299	599
					-	-	-	-	449	
					-	-	-	-	1499	
0.001	58667.85	0	0	1.255	-	-	-	-	2999	5999
					-	-	-	-	4499	

s threshold; and $\ell^{\Delta}(\Delta u_k) \triangleq \Delta u_k^{\top} W_{\Delta u} \Delta u_k \in \mathbb{R}_{\geq 0}$ represents the penalisation of control signal variations $\Delta u_k \triangleq u_k - u_{k-1}$, to extend actuators life and assure a smooth operation. Furthermore, W_e, W_s, $W_{\Delta u}$ are diagonal positive definite matrices that weight each decision variable in their corresponding cost function, and λ_1, λ_2, λ_3 are positive scalar weights that allow to prioritise the impact of each involved objective on the overall performance of the system. These weights are assumed to be fixed by the managers of the DWN. Regarding constraint (2.3), the minimal volume of water required in a tank is given by its net demand, hence, the minimal value of x should be ideally given by $s_k = -B_p d_k$. For the TB-MPC approach, this minimal volume applies for each $j \in \mathbb{Z}_1^{N_s}$ prediction model, i.e., $\bar{x}_i^j \geq -B_p \bar{d}_i^j - \xi_i^j$ for all i prediction steps. In this approach, robustness comes from the fact of considering N_s scenarios simultaneously. In contrast, the deterministic equivalent CC-MPC approach automatically computes a minimal volume given by the net predicted demand plus a robustness factor that depends on the forecast variance and the risk acceptability level as shown in (1.14).

1.3.5 Results

Numeric results of applying the deterministic equivalent CC-MPC and TB-MPC discussed in Section 1.2 to the DWN case study are summarised in Table 1.1. Simulations have been carried out over a time period of eight days, i.e., $n_s = 192$ hours, with a sampling time of one hour. The patterns of the applied demands considered here were synthesised from real values reported for this consumption nodes in the Barcelona DWN between July 23th and July 27th, 2007. Initial conditions, i.e., source capacities, initial volume of water at tanks and starting demands, are set a priori according to real data. The weights of the cost function (1.21) are $\lambda_1 = 100$, $\lambda_2 = 10$, and $\lambda_3 = 1$. The prediction horizon is selected as $N = 24$ hours, due to the periodicity of disturbances. The formulation of the optimisation problems and the closed-loop simulations have been carried out using MATLAB R2012b (64 bits) and CPLEX solver, running in a PC Intel Core E8600 at 3.33GHz with 8GB of RAM.

The key performance indicators used to assess the aforementioned controllers are defined as follows:

$$\Phi_1 \triangleq \frac{24}{n_s}\sum_{k=1}^{n_s} \ell_k, \tag{1.22a}$$

$$\Phi_2 \triangleq |\left\{k \in \mathbb{Z}_1^{n_s} \mid x_k < -B_p d_k\right\}|, \tag{1.22b}$$

$$\Phi_3 \triangleq \sum_{k=1}^{n_s}\sum_{i=1}^{n_x} \max\{0, -B_{p(i)} d_k - x_{k(i)}\}, \tag{1.22c}$$

$$\Phi_4 \triangleq \frac{1}{n_s}\sum_{k=1}^{n_s} t_k, \tag{1.22d}$$

where Φ_1 is the average daily multi-objective cost with ℓ_k given by (1.21), Φ_2 is the number of time instants where water demands are not satisfied (for this, $|\cdot|$ denotes the cardinal of a set of elements), Φ_3 is the accumulated volume of water demand that was not satisfied over the simulation horizon, and Φ_4 is the average time in seconds required to solve the MPC problem at each time instant $k \in \mathbb{Z}_1^{n_s}$. The effect of considering different levels of joint risk acceptability was analysed for the CC-MPC approach, using $\delta = \{0.3, 0.2, 0.1, 0.01, 0.001\}$. In the same way, the TB-MPC approach was applied considering different sizes for the initial set of scenarios, i.e., $N_s = \{19, 28, 59, 599, 5999\}$. The size of this initial set was computed following the bound proposed in [31] taking into account the risk levels involved in the chance constraints. This initial set was reduced later by a factor of 0.25, 0.50, and 0.75 to obtain different rooted trees with N_r scenarios.

As shown in Table 1.1, the different CC-MPC scenarios highlight that reliability and control performance are conflicting objectives, i.e., the inclusion of safety mechanisms in the controller increases the reliability of the DWN in terms of demand satisfaction, but also the cost of its operation. The main advantage of the CC-MPC is its formal methodology, which leads to obtain optimal safety constraints that tackle uncertainties and allow to achieve a specified global service level in the DWN. Moreover, the deterministic equivalent CC-MPC robustness is achieved with a low computational burden given that the only extra load (comparing with a nominal formulation) is the computation of the stochastic characteristics of disturbances propagated in the prediction horizon. In this way, the deterministic equivalent CC-MPC approach is suitable for real-time control of large-scale DWNs. Regarding the TB-MPC approach, numeric results show that considering higher N_s increments, in average, the stage cost while reducing the volume of unsatisfied water demand. Nevertheless, this latter observation is not applicable for the different N_r cases within a same N_s. This might be influenced by the quality of the information that remains after the scenario generation and reduction algorithms that affect the robustness of the approach and will be subject of further research. The main drawback of the TB-MPC approach is the solution average time and the computational burden. In this case study, the implementation for all cases taking $N_s = \{599, 5999\}$ was not possible due to memory issues. Hence, some simplification assumptions as those used in [21] or parallel computing techniques might be useful.

1.4 Conclusions

In this report, two stochastic control approaches have been assessed to deal with the management of a DWN. According to the preliminary results obtained with the considered case study, both techniques show a relatively similar performance. However, it seems clear that CC-MPC is more appropriate when requiring a low probability of constraint violation, because the use of TB-MPC demands the inclusion of a higher number of scenarios, which may be an issue for the application of the latter to large-scale DWNs. Future work should include a more detailed study regarding number of scenarios contained in the tree. Likewise, distributed computation could be used in order to relieve the scaling problems of TB-MPC when the number of scenarios is too high. Finally, the assessment of stochastic techniques should be enhanced with the study of other alternatives.

Chapter 2

Quantifiable Economic Benefits of MPC for DWN

2.1 Introduction

Drinking Water Networks (DWNs) form the link between urban water supply systems and drinking water consumption nodes. These networks are vital for the normal functioning of modern society and maintaining a truly sustainable service is a must in these systems. The management of DWNs involves optimising systematically and simultaneously a collection of (generally conflicting) heterogeneous performance criteria while being subject to different control specifications, constraints and disturbances, which are affected by the dynamism of the economic markets, the social behaviour and the local/regional legislation.

The complexity of DWNs, (*i.e.*, dimensionality, non-linearities, infrastructure constraints, uncertainties), the stronger requirements for water transport service quality and the need for a sustainable exploitation of the available resources make the management of these systems a challenging control problem that has caught the attention of the scientific community, see e.g., [2, 4, 7, 8, 19, 32, 34]. As discussed in the aforementioned references, several approaches are reported in the literature to address the operation of DWNs, ranging from heuristics and expert systems to more advanced mathematical modelling and optimisation techniques, hierarchical-decompositions, combinatorial schemes, and more recently, model predictive control (MPC). This latter one has proven to be one of the most effective and accepted control strategies for large-scale complex systems due to its flexibility to manage constraints and to optimise multi-objective problems as the ones encountered in the management of water systems [24]. The basic idea of MPC is to exploit a model of the network to simulate its future evolution over a prediction horizon and compute an optimal control action (with respect to a predefined cost function) by solving at each time instant an open-loop optimisation problem in a receding horizon fashion [22].

Within the active research on MPC strategies for economic operation of systems, the predominant approach is to consider a hierarchical control structure [33], where standard MPC controllers are designed for tracking operational set-points that are computed usually in an upper layer (i.e., RTO, SSTO) with complex non-linear stationary models and with larger sampling times than the regulatory MPC layer. Nevertheless, as discussed in [12], model inconsistencies, set-point changes, time-varying parameters, disturbances, and time-scale differences may lead the system to unreachable set-points, suboptimal economic performance and feasibility loss. In order to tackle some of the main drawbacks of the typical hierarchical scheme and take more economic profit from the transitory behaviour of the system, some authors have proposed to integrate the economic optimisation within the MPC using either a two-layer approach, see, e.g., [11, 35], or a single-layer approach, i.e., the so called economic MPC (EMPC) [29]. It has been shown in the aforementioned references that closed-loop stability and/or average asymptotic performance can be guaranteed under some form of dissipativity, duality or convexity assumptions. Nevertheless, as shown in [20], the feasibility of the overall problem might be lost if the economic criteria changes (either under the mentioned one-layer or two-layer schemes), therefore, modified single-layer based controllers that inherit the feasibility guarantee of tracking MPC and the optimality of EMPC have been proposed in this latter reference.

Recent studies on control of DWNs are focused on the design of MPC controllers that directly optimise a (non-standard) economic cost function, see e.g., [24, 26], not to obtain steady-state set-points but target trajectories for low-level PID controllers. This latter is the case of interest of this report, where a stationary operation is not beneficial and possible periodicity of time-varying parameters can be exploited for economic profitability. Despite the good practical performance reported in the aforementioned references, stability and feasibility conditions were not considered.

The main contribution of this report is the comparison of several economically-oriented MPC schemes applied to the management of DWNs. Moreover, assuming the water demand and the energy price as periodically time-varying signals, this report uses the results in [29, 36, 18, 20], and adds periodic terminal constraints to the MPC controller developed in [26] to equip it with guarantees of recursive feasibility and convergence.

The report is organized as follows. Section 2.2 describes a control-oriented flow-based model for DWNs and states the control objective. Section 2.3 formulates the MPC strategies to be compared for the economic management of DWN. Section 2.4 describes the case study and presents the results of applying four EMPC schemes. Section 2.5 highlights the concluding remarks that can be drawn from the results presented here as well as some future research directions.

2.2 Problem Statement

2.2.1 Control-Oriented Modelling

This report considers a general DWN being represented by a directed graph $G(\mathcal{V}, \mathcal{E})$, where a set of elements, i.e., n_s sources, n_x storage tanks, n_q intersection nodes, and n_d sinks, are represented by $v \in \mathcal{V}$ vertices, which are connected by $a \in \mathcal{E}$ directed links. Due to the network function, water is transported along the links by n_u flow actuators (i.e., pipes and valves), passing through reservoirs or tanks, from specific origin locations to specific destination locations. The network is subject to several capacity and operational constraints, and to measured stochastic flows to sinks driven by customers water demand.

Setting the volume in the storage tanks as the state variable $x_k \in \mathbb{R}^{n_x}$, the flow through the actuators as the manipulated inputs $u_k \in \mathbb{R}^{n_u}$, and the demanded flow as measured disturbances $d_k \in \mathbb{R}^{n_d}$, then the nominal control-oriented model of the DWN $G(\mathcal{V}, \mathcal{E})$ may be abstracted and described by the following set of linear discrete-time difference-algebraic equations for all time instant $k \in \mathbb{N}$:

$$x_{k+1} = Ax_k + Bu_k + B_d d_k, \tag{2.1a}$$
$$0 = E_u u_k + E_d d_k, \tag{2.1b}$$

where the difference equations in (2.1a) describe the dynamics of the storage tanks, and the algebraic equations in (2.1b) describe the static relations (i.e., mass balance at junction nodes) in the network. Moreover, A, B, B_d, E_u, E_d, are time-invariant matrices of suitable dimensions dictated by the network topology.

The states and control inputs are subject to hard constraints due physical and operational limits, i.e.,

$$x_{\min} \leq x_k \leq x_{\max}, \quad \forall k \in \mathbb{N} \tag{2.2a}$$
$$u_{\min} \leq u_k \leq u_{\max}, \quad \forall k \in \mathbb{N} \tag{2.2b}$$

where $x_{\min}$ and $x_{\max}$ are vectors in $\mathbb{R}^{n_x}$ of the minimum and maximum volume of water that the tanks are able to store, and similarly, $u_{\min}$ and $u_{\max}$ are vectors in $\mathbb{R}^{n_u}$ of the minimum and maximum allowable water flow through actuators. Moreover, for safety and supply service reliability, the states are subject to the following management soft constraint:

$$x_k \geq s - \xi_k, \quad \xi_k \geq 0, \quad \forall k \in \mathbb{N} \tag{2.3}$$

where $s \in \mathbb{R}_+^{n_x}$ is a positive vector of base stocks (minimal volume in each tank to avoid stock-outs) and $\xi_k \in \mathbb{R}_+^{n_x}$ is a vector of positive slack variables to be minimised, which represents the amount of water volume in tanks that is allowed to go below from the desired base stocks.

Assumption 4. *The state x_k and the demand d_k are measured at any time instant k, and the pair (A, B) is controllable. Furthermore, all elements operate with a common review period Δt and all storage tanks are subject to the same replenishment policy.*

Assumption 5. *Water demands are non-stationary and present a cyclic pattern with known period $T \geq 1$, hence, $d_{k+T} = d_k$ for all $k \in \mathbb{N}$. The admissible demands lie in* *a polytopic convex set, i.e., $d_k \in \mathbb{W} \subset \mathbb{R}^{n_d}$ for all $k \in \mathbb{N}$. The periodic trajectory of this exogenous signal, i.e., $\mathbf{d}_k = \{d_i\}_{i=k}^{k+T-1}$, will be assumed as perfectly known when designing the nominal predictive controllers.*

2.2.2 System Management Criteria

The management of a DWN is a multi-objective optimisation problem. In this report, three operational goals with different nature are considered, i.e., economic, safety, and smoothness objectives. Therefore, the control task is to design a control law $u_k = \kappa(x_k, \mathbf{d}_k)$ that minimises the following cost functions:

$$\ell_k^e \triangleq \alpha_k^\top W_e u_k \;, \tag{2.4a}$$
$$\ell_k^s \triangleq \xi_k^\top W_s \xi_k \;, \tag{2.4b}$$
$$\ell_k^\Delta \triangleq \Delta u_k^\top W_{\Delta u} \Delta u_k, \tag{2.4c}$$

where $\ell_k^e \in \mathbb{R}_{\geq 0}$ represents the economic cost of network operation that depends on a time-of-use pricing scheme given by $\alpha_k \triangleq (\alpha_1 + \alpha_{2,k}) \in \mathbb{R}_+^{n_u}$, which takes into account a fixed water production cost $\alpha_1 \in \mathbb{R}_+^{n_u}$ and a time-varying water pumping cost $\alpha_{2,k} \in \mathbb{R}_+^{n_u}$ that changes every time instant k according to the dynamic electric tariff. The cost $\ell_k^s \in \mathbb{R}_{\geq 0}$ penalises the amount of water volume that goes below from the pre-specified security threshold s in (2.3), i.e., $\xi_k = x_k - s$ if $x_k \leq s$, otherwise $\xi_k = 0$. The cost $\ell_k^\Delta \in \mathbb{R}_{\geq 0}$ represents the penalisation of control signal variations $\Delta u_k \triangleq u_k - u_{k-1}$, which assures a smooth operation and extends actuators life. Furthermore, W_e, W_s, $W_{\Delta u}$ are diagonal positive definite matrices that weight each decision variable in their corresponding cost function.

Assumption 6. *The pricing in the economic cost is assumed to be periodic, i.e., $\alpha_k = \alpha_{k+T}$ for all $k \in \mathbb{N}$.*

To achieve the control task, the above predefined objectives are aggregated in a weighted sum stage cost function

$$V_k(x, u) \triangleq \lambda_1 \ell_k^e + \lambda_2 \ell_k^s + \lambda_3 \ell_k^\Delta, \tag{2.5}$$

where λ_1, λ_2, λ_3 are positive scalars that allow to prioritise the impact of each involved objective on the overall performance of the system. Ideally, the resultant control strategy should fill the tanks during the periods of lower energy cost with water taken from the cheapest sources, and deplete them when compensating demands. Given Assumptions 5 and 6, the optimal nominal behaviour of the system for a known period $T \geq 1$ is defined by the T-periodic sequences $\mathbf{x}^\star = \{x_i^\star\}_{i=0}^{T-1}$ and $\mathbf{u}^\star = \{u_i^\star\}_{i=0}^{T-1}$, which result from solving the following finite horizon optimisation problem (FHOP):

$$J_p^\star \triangleq \min_{\mathbf{x}^\star \mathbf{u}^\star \boldsymbol{\xi}^\star} \sum_{k=0}^{T-1} \tilde{V}_k(x_k, u_k) \tag{2.6a}$$

subject to (2.1), (2.2), (2.3) $\forall k \in \mathbb{N}_{[0,T-1]}$ and

$$x_T = x_0, \tag{2.6b}$$

where $\tilde{V}_k(x_k,u_k) \triangleq V_k(x_k,u_k) + \delta\|x_k\|^2$. This last term, i.e., $\delta\|x_k\|^2$ with $\delta \in \mathbb{R}_+$ and sufficiently small, is a convex regularisation term added to enforce the uniqueness of the solution to (2.6) given that the cost function (2.5) is originally not convex with respect to x. Defining the weighting matrices W_s and $W_{\Delta u}$ in (2.4) to be positive definite, then $\tilde{V}_k(x_k,u_k)$ is bounded and strictly convex by design with respect to (x,u,ξ). Hence, (2.6) has a unique solution for a given nominal periodic sequence of water demand $\mathbf{d}_k = \{d_i\}_{i=k}^{k+T-1}$ and economic pricing $\boldsymbol{\alpha}_k = \{\alpha_i\}_{i=k}^{k+T-1}$. Therefore, the equality $(x_k^\star, u_k^\star, d_k) = (x_{k+cT}^\star, u_{k+cT}^\star, d_{k+cT})$ holds for any positive integer $c \geq 0$. These conditions allow to apply different EMPC strategies for the control of DWNs.

2.3 Economic MPC strategies for the management of DWNs

In this section, different EMPC strategies are stated for the DWN control problem in order to be compared. These schemes are: a two-layer architecture where an economic planner and a tracking MPC are interacting, a standard EMPC with terminal state constraint, and two one-layer EMPC strategies that account for changes in the economic criteria.

2.3.1 Hierarchical EMPC

This is a two-layer optimal controller, where a separation of objectives, models and/or time-scales may be performed. Below are stated the optimisation problems involved in this hierarchical approach.

Upper layer EMPC

In this layer, an RTO is used to compute at the beginning of each operating cycle the optimal time-varying state and input trajectories using current measurements. The associated FHOP is stated as

$$\min_{\mathbf{x}_z^r \mathbf{u}_z^r \boldsymbol{\xi}_z^r} \sum_{k=z}^{z+H_p^u-1} \tilde{V}_k(x_k,u_k) \tag{2.7a}$$

subject to (2.1), (2.2), (2.3) $\forall k \in \mathbb{N}_{[z,z+H_p^u-1]}$ and

$$x_{z+H_p^u} = x_z, \quad x_z = \bar{x}_z, \tag{2.7b}$$

where $H_p^u \in \mathbb{N}$ is the prediction horizon, $\bar{x}_z \in \mathbb{R}^{n_x}$ is the measured initial state at time instant $z \in \mathbb{N}$, and $\mathbf{x}_z^{r\star} \triangleq \{x_k^\star\}_{k=z}^{z+H_p^u-1}$ and $\mathbf{u}_z^{r\star} \triangleq \{u_k^\star\}_{k=z}^{z+H_p^u-1}$ are the optimal state and input trajectories calculated from (2.7) to govern the lower layer MPC. Defining, $x_k^r \triangleq x_k^\star$ and $u_k^r \triangleq u_k^\star$ for $k \in \mathbb{N}_{[z,z+H_p^u-1]}$, a lower layer MPC is designed as follows.

Lower layer Tracking MPC

In this layer, a conventional MPC is used to enforce the system to track the pre-computed optimal trajectories. The associated FHOP is stated as

$$\min_{\mathbf{u}_t \boldsymbol{\xi}_t} \sum_{k=t}^{t+H_p^l-1} \|x_k - x_k^r\|_{Q_x}^2 + \|u_k - u_k^r\|_{Q_u}^2 \tag{2.8a}$$

subject to (2.1), (2.2), (2.3) $\forall k \in \mathbb{N}_{[t,t+H_p^l-1]}$ and

$$x_{t+H_p^l} = x_{t+H_p^l}^r, \quad x_t = \bar{x}_t, \tag{2.8b}$$

where $H_p^l \in \mathbb{N}$ is the prediction horizon of the lower layer, $\bar{x}_t \in \mathbb{R}^{n_x}$ is the measured initial state at time instant $t \in \mathbb{N}$, and Q_x and Q_u are appropriate positive definite weighting matrices. Following the receding horizon technique, the control law derived under this hierarchical scheme is given by $\kappa(\bar{x}_t, \mathbf{d}_t) = u_t^\star(\bar{x}_t, \mathbf{d}_t)$, i.e., only the first control action of the optimal input sequence obtained in (2.8) is applied to the system. If asymptotic convergence to the upper layer trajectory is desired, the tracking problem can be reformulated in terms of the error $e_k = x_k - x_k^r$, leading to a set-point (origin) stabilization problem of the error dynamics; see, e.g., [13].

Note that the prediction horizon H_p^u of the upper layer must be large enough to cover the nominal operating cycle T and the prediction horizon H_p^l of the lower layer, i.e., $H_p^u \geq T + H_p^l$. Furthermore, the upper layer may have an equal or larger sampling time than the one of the lower layer, i.e., $\Delta t_1 \geq \Delta t_2$. The main drawback of this two-layer MPC approach for the management of DWNs is that if the energy price schedule or the water demand pattern changes in time with a high rate, then the transitory periods will be so that the interaction between layers will lead to a possible loss of feasibility or to an economic performance degradation.

2.3.2 Standard EMPC

The main feature of this approach, in contrast with the hierarchical scheme, is that even when the two-layers may work with the same sampling time, the standard EMPC considers the global cost function directly as the stage cost of the controller objective and avoids penalising the tracking error to the targets. The associated FHOP for the periodic operation of DWNs is stated as

$$\min_{\mathbf{u}_t,\boldsymbol{\xi}_t} \sum_{k=t}^{t+H_p-1} \tilde{V}_k(x_k,u_k) \tag{2.9a}$$

subject to (2.1), (2.2), (2.3) $\forall k \in \mathbb{N}_{[t,t+H_p-1]}$ and

$$x_{t+H_p} = x^{\star}_{\mathrm{mod}(t+H_p,T)}, \quad x_t = \bar{x}_t, \tag{2.9b}$$

where $\bar{x}_t \in \mathbb{R}^{n_x}$ is the measured initial state at time instant $t \in \mathbb{N}$, and $x^{\star}_{\mathrm{mod}(t+H_p,T)}$ is the optimal periodic value obtained in (2.6) that corresponds to the time instant t. As shown in [1], the standard EMPC is capable of enhancing the economic performance of the system and achieving an asymptotic average cost, which is at least as good as that of the best periodic trajectory. Moreover, convergence can be enforced with the EMPC strategy if the cost function is modified to be dissipative with respect to the optimal periodic trajectory (by adding, e.g., convex regularization terms as done in [1]). The stability of the closed-loop system around the feasible optimal operating cycle can be guaranteed following the results in [36] for periodic systems.

Even when this EMPC controller improves the average economic performance of the DWN, its main weakness is also a possible loss of feasibility, especially if the parameters affecting the cost function, i.e., water and energy prices or priority weights, change the optimal cycle or the target state to unreachable values for the given prediction horizon.

2.3.3 EMPC for DWNs with Changing Operating Patterns

In order to overcome the possible loss of feasibility due to changing operating patterns caused by the parameters of the cost function or by the demands, two approches following the ideas in [18] and [20] are here proposed to be solved in a one-layer architecture. These schemes integrate in different ways the optimal trajectory problem (2.6) with the standard EMPC in (2.9).

Option A: Enlargement of the prediction horizon

$$\min_{\mathbf{u}_t,\boldsymbol{\xi}_t} \sum_{k=t}^{t+H_p+T-1} \tilde{V}_k(x_k,u_k) \tag{2.10a}$$

subject to (2.1), (2.2), (2.3), and

$$x_{k+H_p+T} = x_{k+H_p}, \quad x_t = \bar{x}_t. \tag{2.10b}$$

Note that in this option, slight changes of the EMPC framework are required, i.e., the terminal constraint in (2.10b) is associated to a periodic trajectory that results from the same prediction model used in the optimisation problem rather than to a precomputed trajectory, and the prediction horizon is extended to cover the period of the process dynamics.

Option B: Inclusion of a pseudo-reference to track

$$\min_{\mathbf{u}_k \mathbf{x}^s_k, \mathbf{u}^s_k, \boldsymbol{\xi}_k} \sum_{k=t}^{t+H^s_p-1} \gamma_{\mathrm{O}} \tilde{V}_k(x^s_k, u^s_k) + \sum_{k=t}^{t+H_p-1} \gamma_{\mathrm{T}} \left(\|x_k - x^s_k\|^2_{Q_x} + \|u_k - u^s_k\|^2_{Q_u} \right) \tag{2.11a}$$

subject to (2.1), (2.2), (2.3) $\forall k \in \mathbb{N}_{[t,t+H_p]}$ and

$$x^s_{k+1} = Ax^s_k + Bu^s_k + B_d d_{t+k}, \quad \forall k \in \mathbb{N}_{[t,t+H^s_p-1]} \tag{2.11b}$$

$$E_u u^s_k + E_d d_{t+k} = 0, \quad \forall k \in \mathbb{N}_{[t,t+H^s_p-1]} \tag{2.11c}$$

$$\epsilon x_{\min} \leq x^s_k \leq \epsilon x_{\max}, \quad \forall k \in \mathbb{N}_{[t,t+H^s_p]} \tag{2.11d}$$

$$\epsilon u_{\min} \leq u^s_k \leq \epsilon u_{\max}, \quad \forall k \in \mathbb{N}_{[t,t+H^s_p]} \tag{2.11e}$$

$$x_k \geq s - \xi_k, \quad \xi_k \geq 0, \quad \forall k \in \mathbb{N}_{[t,t+H^s_p]} \tag{2.11f}$$

$$x^s_{t+H^s_p} = x^s_t, \tag{2.11g}$$

$$x_{t+H_p} = x^s_{t+H_p}, \tag{2.11h}$$

$$x_t = \bar{x}_t, \tag{2.11i}$$

where Q_x and Q_u are positive definite matrices, γ_{O} and γ_{T} are positive scalars introduced to establish a trade-off between economic and tracking performance, and $\epsilon \in (0, 1)$ is a tightening factor included to avoid active constraints at the optimal pseudo-reference. The prediction horizons should be selected such that $H^s_p \geq H_p$.

Remark 2. *Controllers* (2.10) *and* (2.11) *are not enforcing convergence to the precomputed optimal trajectory obtained in* (2.6). *Instead, they are meant to retain feasibility under possible changes of the economic parameters in the cost function and to find new optimal trajectories for the current conditions. To guarantee asymptotic stability to the trajectory obtained in* (2.6), *either when using* (2.10) *or* (2.11), *the results in [18] and [20] can be applied, respectively, considering the error dynamics (regarding the target periodic trajectory) under the assumptions of controllability of the system and the boundedness and convexity of the constraints and cost function involved.*

2.4 Case Study Analysis and Discussion

2.4.1 Description

This section briefly describes a motivational example useful to develop EMPC approaches to solve multi-objective constrained optimisation problems that may appear in the control of DWNs. In general, the DWN operation is driven by the energy prices and the exogenous and endogenous demands. The system under study is a portion extracted from the Barcelona DWN reported in [24]. In Fig. 1.1, a graphical representation of the DWN example is shown, which contains 2 water sources, 3 tanks, 6 manipulated actuators, 4 demand sectors and 2 intersection nodes.

The matrices and vectors that define the system and constraints are the following:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \Delta t,$$

$$B_d = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \Delta t,$$

$$E = \begin{bmatrix} 1 & -1 & -1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 & 0 \end{bmatrix}, \; E_d = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix},$$

$$x_{\min} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^\top,$$

$$x_{\max} = \begin{bmatrix} 470 & 960 & 3100 \end{bmatrix}^\top,$$

$$u_{\min} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^\top,$$

$$u_{\max} = \begin{bmatrix} 1.2970 & 0.0500 & 0.1200 & 0.0150 & 0.0317 & 0.0220 \end{bmatrix}^\top.$$

2.4.2 Results

This section presents the results of applying the EMPC approaches described in Section 2.3 to the aforementioned case study. The sampling time is $\Delta t = 3600$ seconds. The simulation horizon is sixteen days ($N = 384$ hours) for each strategy. The weights of the aggregate user-defined cost function are $\lambda_1 = 100$, $\lambda_2 = 10$, and $\lambda_3 = 0.005$. For the tracking terms, the weighting matrices (Q_x and Q_u) are set up as identity matrices of proper dimensions. The prediction horizon has been selected as $H_p = 24$h, due to the periodicity of both water demands and electricity prices. For the hierarchical controller, the upper layer is executed every 24h as usually done in water distribution scheduling, while the lower layer runs in an hourly basis as in the other economic MPC strategies. The initial common state for all simulations is $x_0^\star = [160.44, 646.23, 633.89]^\top$ in m^3 and the security threshold is $s = [42, 18.0, 270]^\top$ in m^3. The simulations have been carried out using CPLEX 12.5 and Matlab® R2010b (64 bits), running in a PC Intel® Core™ E8600 at 3.33GHz with 8GB of RAM. The closed-loop performance of each controller

Table 2.1: Comparison of controller performance

Controller	KPI$_E$	KPI$_{\Delta U}$
Ideal scheduling (Problem (2.6))	28.6056	2.06×10^{-3}
HEMPC$_{(1)}$	28.4347	2.14×10^{-3}
HEMPC$_{(2)}$	28.6114	2.08×10^{-3}
EMPC$_{(1)}$	28.4124	8.89×10^{-7}
EMPC$_{(2)}$	28.6080	8.89×10^{-7}
EMPCT-A$_{(1)}$	28.4124	8.89×10^{-7}
EMPCT-A$_{(2)}$	28.6041	8.89×10^{-7}
EMPCT-B$_{(1)@\{\gamma_{\mathrm{O}}=1,\, \gamma_{\mathrm{T}}=1\}}$	28.5165	2.17×10^{-3}
EMPCT-B$_{(1)@\{\gamma_{\mathrm{O}}=1,\, \gamma_{\mathrm{T}}=10\}}$	28.4493	2.35×10^{-3}
EMPCT-B$_{(1)@\{\gamma_{\mathrm{O}}=1,\, \gamma_{\mathrm{T}}=100\}}$	28.4178	2.34×10^{-3}
EMPCT-B$_{(1)@\{\gamma_{\mathrm{O}}=10,\, \gamma_{\mathrm{T}}=1\}}$	28.8128	2.29×10^{-3}
EMPCT-B$_{(2)@\{\gamma_{\mathrm{O}}=1,\, \gamma_{\mathrm{T}}=100\}}$	28.6120	2.43×10^{-3}

Subindex (1) indicates non-periodic behaviour enforced while (2) indicates that the periodic constraint is enforced.

has been assessed using the following average key performance indicators (KPI) for the economic, safety and smoothness objectives, respectively:

$$\mathrm{KPI}_E \triangleq \frac{1}{N}\sum_{k=1}^{N}(\alpha_1 + \alpha_{2,k})^\top |\mathbf{u}_k| \Delta t, \tag{2.12}$$

$$\mathrm{KPI}_S \triangleq \frac{1}{N}\sum_{k=1}^{N}\sum_{i=1}^{n_x} \xi_{(i),k}, \tag{2.13}$$

$$\mathrm{KPI}_{\Delta U} \triangleq \frac{1}{N}\sum_{k=1}^{N} \|\Delta u_k\|^2. \tag{2.14}$$

Results are summarised in Table 2.1. The safety indicator has been omitted in this table given that, for all simulated scenarios and strategies, KPI$_S = 0$, which means that all of the MPC controllers decided not to use water from the safety-stocks for the given periodic demand. Note that for each strategy the enforcement of terminal constraints implies an increment of the economic cost. This decrease in performance is the price for gaining in stability. Furthermore, Table 2.2 discloses details of the production and operational costs related to each strategy starting from a non-optimal state $x_0 = [92.45, 905.82, 504.14]^\top$ in m^3, and compares the daily average economic performance ($\mathrm{DAP} = 24 \times \mathrm{KPI}_E$) of the controllers enforcing their corresponding periodic terminal constraints. For the standard EMPC$_{(2)}$, the terminal constraint is set up in relation to a pre-calculated optimal cycle obtained from (2.6). In the HEMPC$_{(2)}$, the

Table 2.2: Comparison of daily average costs of EMPC strategies

Controller	Water Cost (e.u./day)	Electric Cost (e.u./day)	Daily Cost (e.u./day)
$\text{EMPC}_{(2)}$	577.24	110.04	687.28
$\text{HEMPC}_{(2)}$	610.02	134.13	744.15
$\text{EMPCT-A}_{(2)}$	577.79	109.56	687.35
$\text{EMPCT-B}_{(2)}$	577.75	109.79	687.54

e.u.: economic units.

reference trajectory is computed by the upper layer every 24h. For controllers EMPCT-$\text{A}_{(2)}$ and EMPCT-$\text{B}_{(2)}$, no pre-calculated trajectory is needed. It can be seen how the $\text{HEMPC}_{(2)}$ cost degrades notoriously the performance in comparison with the other MPC strategies due to the time-scale separation in its layers. Even when feasibility issues were not found for any of the strategies in this case study, these results reaffirm the current tendency of improving the economic performance by migrating to one-layer EMPC which are robust to changes in the cost function.

In order to further highlight the performance of the EMPC controllers described in Section 2.3.3 that copes with changing economic criterion, the price parameter of the economic term in (2.4) has been affected switching the price profile at different time instants but keeping the same period, see Fig. 2.2. As it can be seen in Fig. 2.1 for the evolution of the states, both controllers maintain the recursive feasibility and stabilise at similar trajectories. Even so, the approach in (2.11), which includes a pseudo reference and tracking terms, presents a slightly higher cost with respect to the approach in (2.10). This behaviour might be due to the regularisation terms that decrease the economic performance if design parameters are not properly tuned.

2.5 Conclusions

Here, the potential of economic MPC for the management of DWNs has been verified on a proof of concept case study. A multi-objective cost function was considered and different EMPC formulations were analysed and extended for controlling a supply water nominal system, where water demands were considered periodic and perfectly known. Especially, the single-layer EMPC approaches resulted to be of great utility for the control of DWNs due to their capacity to cope with changes in the economic parameters of the cost function. Future research lines are the stochastic EMPC formulations to enhance robustness with the minimum performance degradation, the tuning of the EMPC that affects the DWN performance, and the extension of the feasibility, stability and convergence results when dealing with large-scale systems decomposed for decentralised or distributed control.

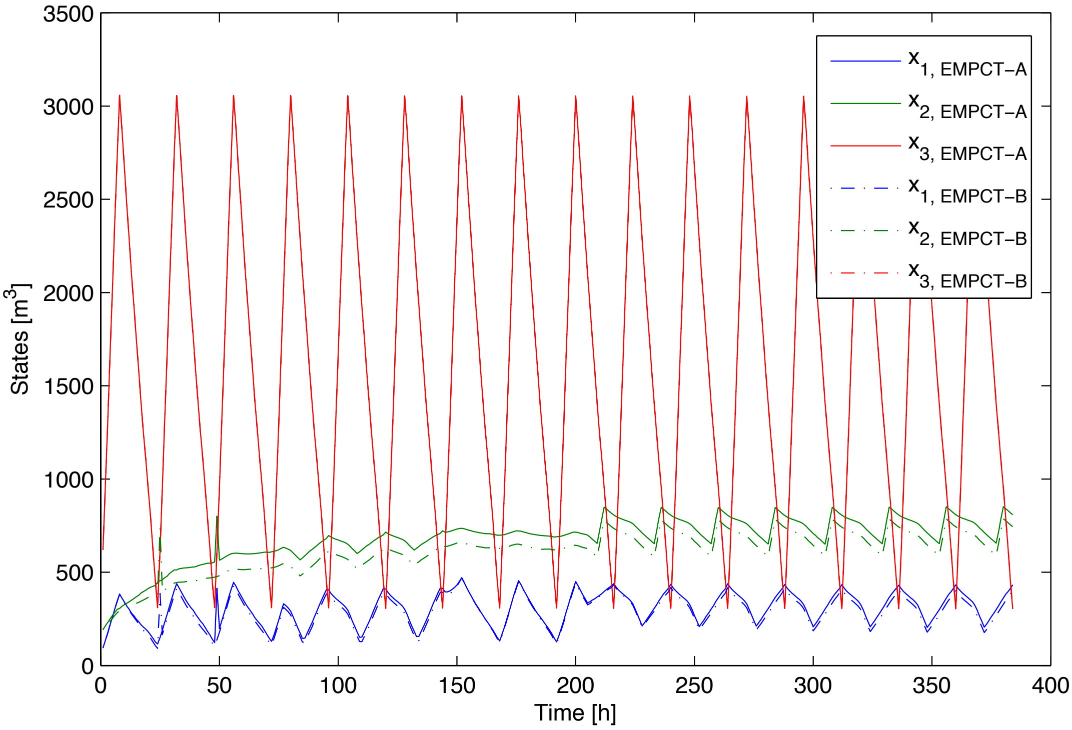

Figure 2.1: Evolution of some states under varying economic parameters

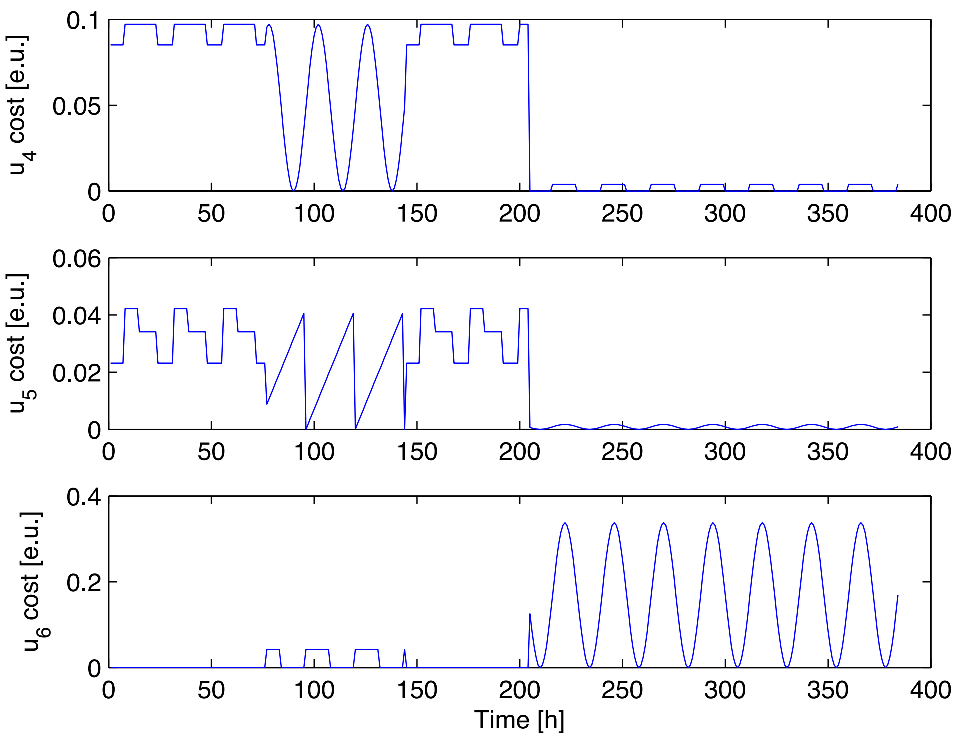

Figure 2.2: Price-of-use for actuators u_4, u_5 and u_6 in economic units (e.u)

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